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Analysis of Stress Concentration in Orthotropic Laminates

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Abstract

The current paper presents a comprehensive finite element analysis on stress concentration in orthotropic laminates. This research investigates the Stress Concentration Factor (SCF) in isotropic and orthotropic plates due to presence of a central cutout with respect to ply orientation angle (θ). The numerical simulations are carried out by using an 8 node shell element in ANSYS APDL environment. It is established from the simulations that the SCF is dependent upon the orientation of the fibers. For orthotropic laminates, SCF depends on material properties. It increases with increase in E_y/E_x ratio and decreases with increase in μ_{xy} .

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1. Introduction

Stress concentrations around cutouts are important from design point of view because they are generally the prime cause of failure under static loads. For most materials, the failure strength is strongly notch (or hole) sensitive [1]. It has been experimentally validated that the net failure stress (considering the reduction in cross-sectional area due to cutouts) is generally much less than the ultimate tensile strength without any discontinuity. For example, strength reductions of 40–60% have been reported for a glass fiber reinforced plastic plate [2]. Failure can be successfully predicted by using elastic stress concentration factor (SCF) without considering sharp edge cracks around the hole. The ratio of the maximum stress at the cutout edge to the nominal stress is called the stress concentration factor (SCF). The stress distribution around an elliptical cut-out in an infinite plate under axial uniformly distributed load was calculated theoretically by Inglis [3] and Kolosoff [4]. Neuber [5] developed an approximate theoretical method which permits the determination of the value of the maximum stress in a finite plate having a central elliptical cut-out subjected to the same loading. Petersons [6] studied the unexpected variations in component geometry of isotropic material subject to static loading and reported its effect on design of machine

component. Simha and Mahapatra analytically studied the stress concentration caused by irregular holes [7]. The effect of dynamic and static loading on stress concentration for a rectangular plate was studied by Zirka et al. [8]. Both orthotropic and isotropic plates were studied by them using photo elastic method. Tafreshi [9] used FEM (Finite Element Method) and BEM (Boundary Element Method) for stress analysis of thick flat plates having oblique cut-outs under uniaxial tension and out-of-plane bending. Kumar et al. [10] did a parametric study on several plate slenderness ratios and by changing the area ratio of cut out to plate to study the influence of ultimate strength on the size of cut-out. It was found that when the area ratio along the loading direction is increased the ultimate strength decreases. Kalita et al. [11-14] has studied the variation of deflection and induced stresses due to presence of central cut-outs under transverse loading. Darwish et al. [15] used ANSYS to study SCF in orthotropic plates with countersunk rivet hole and reported that maximum SCF occurs at countersunk edge.

2. Problem Description

The objective of the current work is to calculate the stress concentration factor for different stresses for laminated orthotropic plate. The influence of fiber orientation is studied for three different orthotropic materials. The various elastic constants of the three orthotropic materials in consideration are shown in Table 1. The orthotropic plate is modeled as an infinite plate. The various dimensions of the modeled plate are shown in Fig. 1. Three ply laminate of each laminate layer 5 mm is considered. In the current paper two types of loading- uniaxial pressure loading of 1MPa is considered in X-direction and biaxial loading in X-direction and Y-direction of 1 Mpa. Shell 281 element from ANSYS APDL library is selected for building the finite element model. It is an 8 node shell element having six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes. So in total each element has 48 degrees of freedom. It is well suited for linear, large rotation and large strain nonlinear applications. It is a good practice and efficient way to exploit symmetry in FEM analysis whenever possible. Hence in this case, only one quadrant of the plate is modeled and meshed. Mapped meshing using concatenate command is exploited to get finer mesh near the discontinuity. This finer mesh near the cutout is well equipped in capturing the SCF in this region.

Table 1. Material Properties considered for the study.

	E_x	E_y	E_z	G_{xy}	G_{xz}	G_{yz}	μ_{xy}	μ_{xz}	μ_{yz}
(in GPa)									
Material#1	44.7	17.9	17.9	8.96	8.96	3.45	0.25	0.25	0.34
Material#2	39	8.6	8.6	3.8	3.8	3.8	0.28	0.28	0.28
Material#3	121	112	121	44	44	44	0.20	0.20	0.20

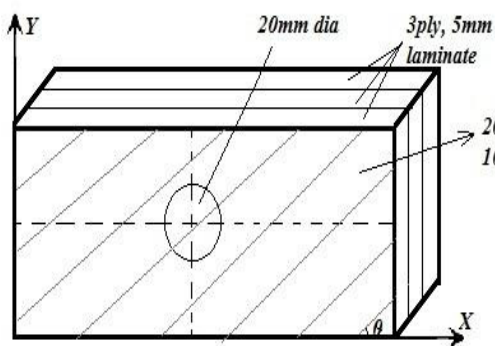


Fig. 1. Basic model of problem.

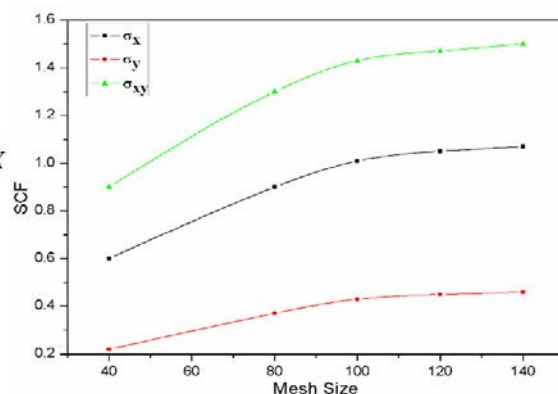


Fig. 2. Mesh convergence study.

A mesh convergence study is carried out to select the best possible mesh size to optimize the FE solution. The results of mesh refinement study is shown in Fig. 2. The various parameters considered in the mesh refinement study are: 3 ply rectangular plate 200mm X 100mm, central circular cut-out 20mm diameter; material 1; $\theta=0^\circ$.

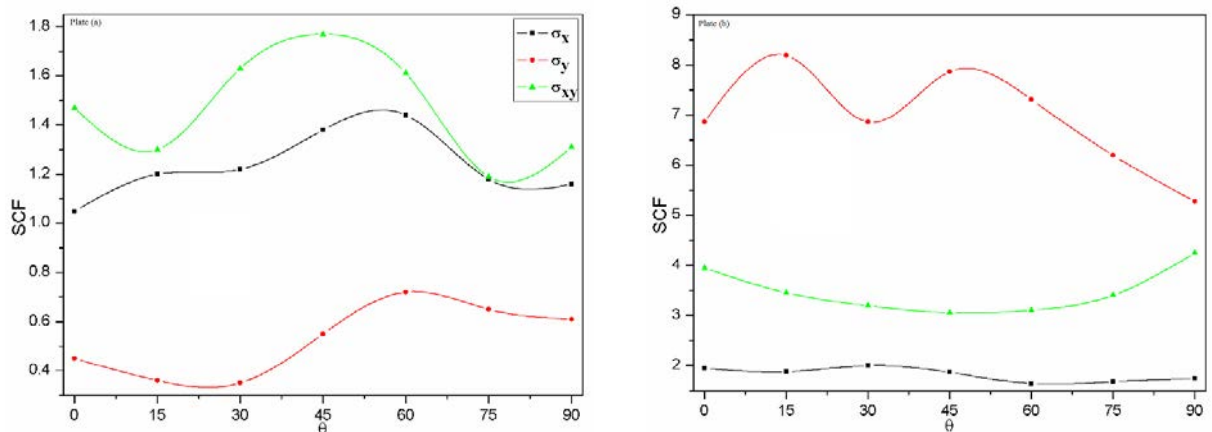


Fig. 3. Effect of orientation angle Θ on SCF for σ_x , σ_y , σ_{xy} of orthotropic material #1 under uniaxial loading.

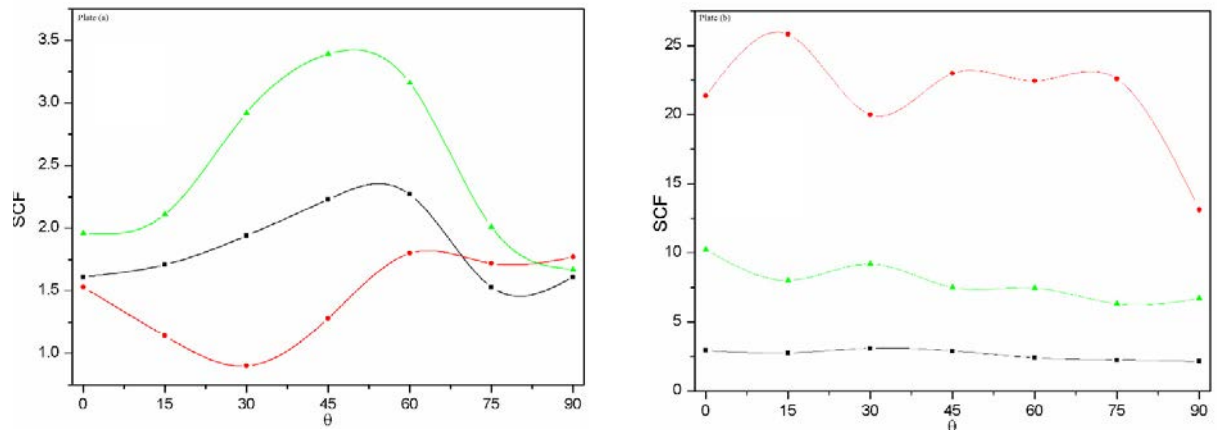


Fig. 4. Effect of orientation angle Θ on SCF for σ_x , σ_y , σ_{xy} of orthotropic material #2 under uniaxial loading.

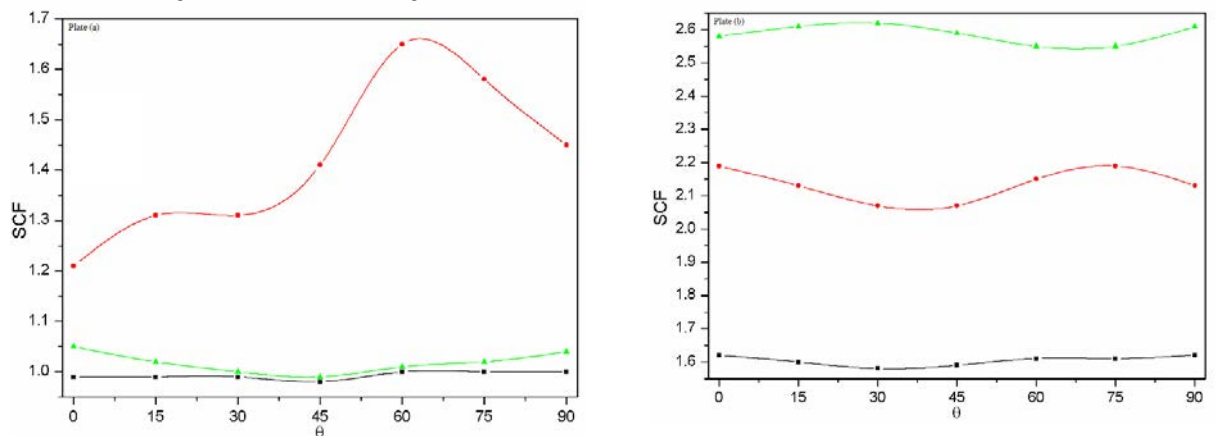


Fig. 5. Effect of orientation angle Θ on SCF for σ_x , σ_y , σ_{xy} of orthotropic material #3 under uniaxial loading.

For each mesh, SCF was obtained and compared with the next finer mesh. Mesh size here indicates the number of elements used per edge in the X and Y edges. For example, if the upper right quadrant is considered for the study a mesh size of 80 would mean the X and Y edges away from the hole would have 80 divisions each. At mesh size 120 the SCF has reached a convergence value with an error about 3% from the asymptote. This error is very small and computation time required is optimum; accordingly mesh 120 was considered throughout the analysis of the present FE model. An ANSYS APDL code file is generated to conduct the numerical parametric experiments.

3. Results and Discussion

3.1. Uniaxial Loading

Numerical results are presented here for SCF for different stresses with change in fiber orientation angles in three orthotropic materials. Two different boundary conditions are considered here- simply supported plate (named here as Plate (a)) and a cantilever plate (named here as plate (b)).

Variation of SCF for σ_x , σ_y , σ_{xy} in case of orthotropic material#1 is shown in Fig. 3 for plate (a) and (b). It is seen that in plate (a) SCF is lowest for σ_y and maximum for σ_{xy} whereas for plate (b) maximum SCF is for σ_y and minimum SCF is obtained for σ_x . σ_y in plate (a) is around 0.45 at $\theta=0^\circ$, it decreases till 30° after which it starts increasing and attains a peak at $\theta=60^\circ$ beyond which there is marginal decrease in SCF for σ_y . The maximum SCF σ_x for plate (a) is at $\theta=60^\circ$ and minimum at $\theta=0^\circ$. The SCF σ_x increases rapidly from $\theta=0^\circ$ to $\theta=60^\circ$ beyond which it starts to decrease till $\theta=90^\circ$. The SCF σ_{xy} is around 1.5 at $\theta=0^\circ$, it decreases till $\theta=15^\circ$ after which it increases to obtain a peak SCF at $\theta=45^\circ$ beyond which it again starts decreasing and minimum SCF for σ_{xy} in case of plate (a) is seen at $\theta=75^\circ$.

In case of plate (b) maximum SCF is seen in case of σ_y and minimum SCF is obtained for σ_x . σ_x is around 2 for $\theta=0^\circ$ and marginal variation in σ_x is seen for plate (b). σ_{xy} is around 4 for $\theta=0^\circ$ after which it starts reducing till $\theta=45^\circ$, at this point it is lowest after which it again starts to increase. The SCF σ_y in case of plate (b) is around 7 at $\theta=0^\circ$ and has a peak value of around 7.5 at $\theta=15^\circ$, beyond $\theta=45^\circ$ it decreases and is minimum at $\theta=90^\circ$. It is also observed that in general the SCF values of different stresses are more in plate (b) are much more as compared to the corresponding values in plate (a).

Variation of SCF for σ_x , σ_y , σ_{xy} in case of orthotropic material#2 is shown in Fig. 4 for plate (a) and (b). It is seen that in plate (a) SCF is lowest for σ_y and maximum for σ_{xy} whereas for plate (b) maximum SCF is for σ_y and minimum SCF is obtained for σ_x . SCF for σ_x is 1.6 at $\theta=0^\circ$ after which it starts increasing till $\theta=45^\circ$ beyond which it decreases till $\theta=75^\circ$ and is constant thereafter. SCF for σ_y is 1.5 at $\theta=0^\circ$ and it decreases till at $\theta=30^\circ$ after which it increases till at $\theta=60^\circ$. Beyond this point the SCF σ_y remains constant. The SCF σ_{xy} is 2 at $\theta=0^\circ$ and it increases till at $\theta=45^\circ$ where SCF is around 3.4 and beyond this point the SCF decreases steeply till at $\theta=90^\circ$.

Fig. 5 shows the variation of SCF for σ_x , σ_y , σ_{xy} in case of orthotropic material#3 for plate (a) and (b). It is observed that the SCF for σ_x is lowest in plate (a) and plate (b) and SCF for σ_y is maximum in plate (a) whereas SCF for σ_{xy} is maximum in plate (b). In plate (a) SCF for σ_x and σ_{xy} is around 1 for all fiber orientation angles and SCF for σ_y is around 1.2 at $\theta=0^\circ$ after which it starts increasing till $\theta=60^\circ$ and beyond this point it decreases till $\theta=90^\circ$.

In case of Plate (b) for orthotropic material#3 it is seen that there is nominal variation in SCF for σ_x , σ_y , σ_{xy} . The SCF for σ_x is constant at around 1.6 and SCF for σ_y is 2.1 and for σ_{xy} the SCF is around 2.6. This is due to the fact that in material#3 the elastic constants ($\frac{E_x}{E_y} = 1.08$) were defined in such a way they somewhat resemble an isotropic material.

3.2. Biaxial Loading

Variation of SCF for σ_x , σ_y , σ_{xy} in case of orthotropic material#1 under biaxial loading is shown in Fig. 6 for plate (a) and (b). It is seen that in plate (a) SCF is lowest for σ_{xy} and maximum for σ_y whereas for plate (b) maximum SCF is for σ_x and minimum SCF is obtained for σ_{xy} . σ_y in plate (a) is around 2.2 at $\theta=0^\circ$, it increases till 30° after which it starts decreasing and minimum at $\theta=75^\circ$ beyond which there is steep increase in SCF σ_y . The maximum SCF σ_x for plate (a) is at $\theta=60^\circ$ and minimum is at $\theta=90^\circ$.

The SCF σ_X increases from $\theta=0^\circ$ to $\theta=15^\circ$ beyond which it starts to decrease till $\theta=45^\circ$. The SCF σ_{XY} is around 1.2 at $\theta=0^\circ$, it decreases till $\theta=30^\circ$ after which it increases till $\theta=60^\circ$ beyond which it again starts decreasing till $\theta=75^\circ$ after which it increases till $\theta=90^\circ$. Minimum SCF σ_{XY} in case of plate (a) is seen at $\theta=75^\circ$ which is similar like in case of uniaxial load.

In case of plate (b) maximum SCF is seen in case of σ_X and minimum SCF is obtained for σ_{XY} . σ_X is around 2.1 for $\theta=0^\circ$ and marginal variation in SCF σ_X is seen for plate (b) till $\theta=30^\circ$. σ_{XY} is around 0.93 for $\theta=0^\circ$ after which it starts reducing till $\theta=60^\circ$, at this point it is lowest after which it again starts to increase. The SCF σ_Y in case of plate (b) is around 2 at $\theta=0^\circ$ and has a peak value of around 2.1 at $\theta=15^\circ$, beyond $\theta=45^\circ$ it decreases and is minimum at $\theta=75^\circ$. It is also observed that in general the SCF σ_X are more in plate (b) are much more as compared to the corresponding values in plate (a) whereas SCF σ_Y and SCF σ_{XY} are more prominent in plate (a).

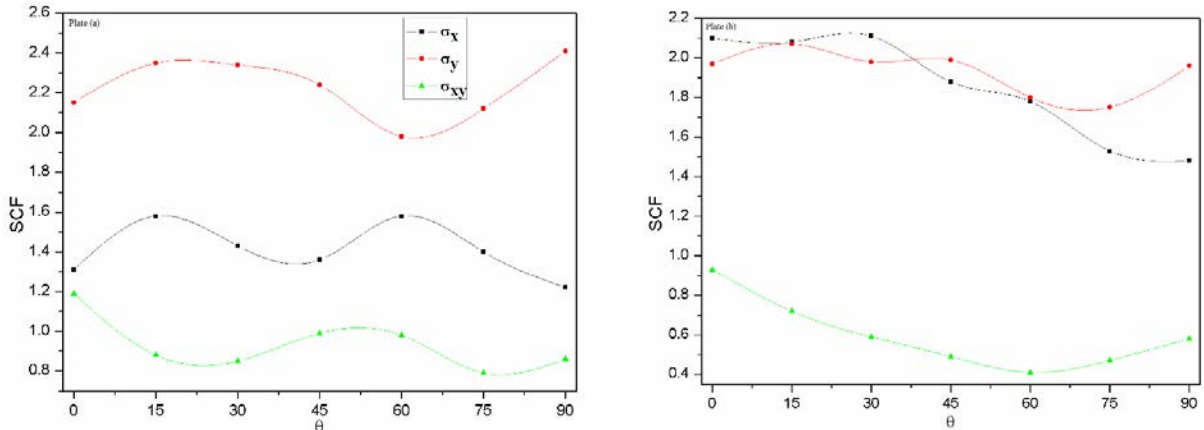


Fig. 6. Effect of orientation angle Θ on SCF for σ_X , σ_Y , σ_{XY} of orthotropic material #1 under biaxial loading.

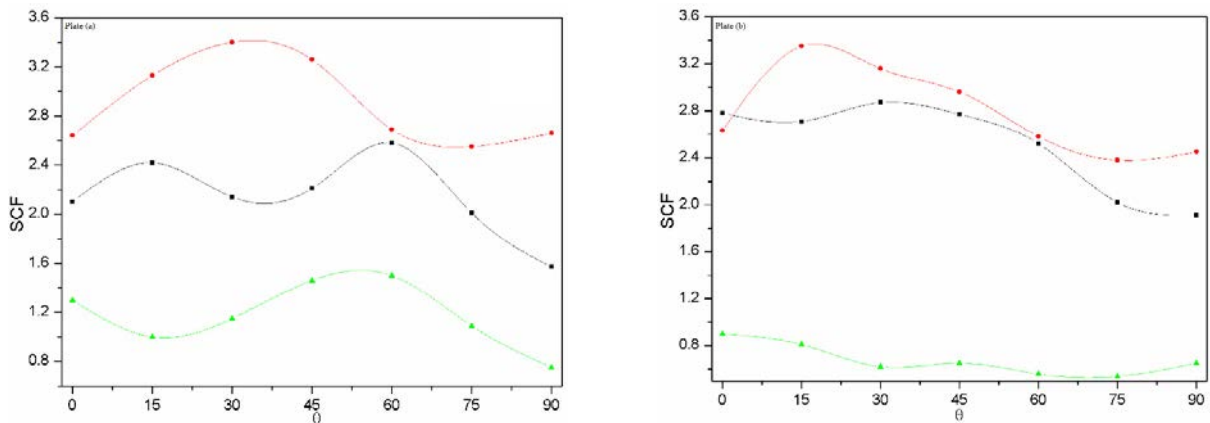


Fig. 7. Effect of orientation angle Θ on SCF for σ_X , σ_Y , σ_{XY} of orthotropic material #2 under biaxial loading.

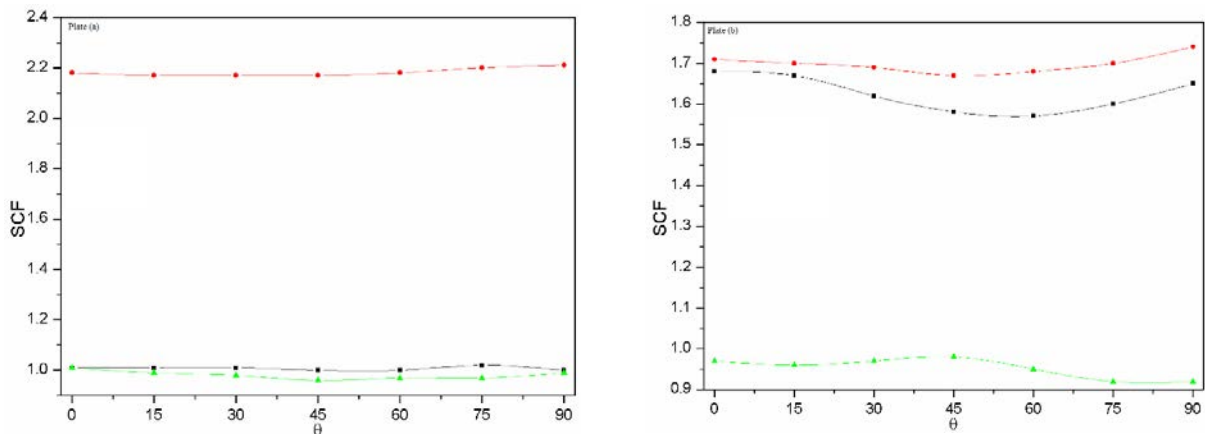


Fig. 8. Effect of orientation angle Θ on SCF for σ_x , σ_y , σ_{xy} of orthotropic material #3 under biaxial loading.

Variation of SCF for σ_x , σ_y , σ_{xy} in case of orthotropic material#2 is shown in Fig. 7 for plate (a) and (b) for biaxial loads. It is seen that in plate (a) SCF is lowest for σ_{xy} and maximum for σ_y whereas for plate (b) maximum SCF is for σ_y and minimum SCF is obtained for σ_{xy} . In plate (a) SCF for σ_x is 2.1 at $\theta=0^\circ$ after which it starts increasing till $\theta=15^\circ$ beyond which it decreases till $\theta=90^\circ$ to attain a minimum value of 1.57. SCF for σ_y is 2.64 at $\theta=0^\circ$ and it increases till at $\theta=45^\circ$ after which it decreases till at $\theta=75^\circ$. The SCF σ_{xy} is 1.3 at $\theta=0^\circ$ and it has a minimum value of 0.75 at $\theta=90^\circ$.

Fig. 8 shows the variation of SCF for σ_x , σ_y , σ_{xy} in case of orthotropic material#3 for plate (a) and (b). It is observed that the SCF for σ_{xy} is lowest in plate (a) and plate (b) and SCF for σ_y is maximum in plate (a) whereas SCF for σ_y is maximum in plate (b). In plate (a) SCF for σ_x and σ_{xy} is around 1 for all fiber orientation angles and SCF for σ_y is around 2.2 at $\theta=0^\circ$ after which it starts increasing till $\theta=90^\circ$.

In case of Plate (b) for orthotropic material#3 it is seen that there is nominal variation in SCF for σ_x , σ_y , σ_{xy} . The SCF for σ_x is constant at around 1.6 and SCF for σ_y is 1.7 and for σ_{xy} the SCF is around 1.

4. Conclusion

The following conclusions can be drawn from the above results

- For both simply supported as well as cantilever plates the SCF for shear stresses are much more as compared to the SCF for normal stresses in X direction under uniaxial loads.
- It is seen that SCF for σ_x , σ_y , σ_{xy} is generally much more in cantilever plate as compared to simply supported plate for uniaxial load.
- SCF for σ_x , σ_y , σ_{xy} is maximum at $\theta=60^\circ$, $\theta=60^\circ$ and $\theta=45^\circ$ respectively for simply supported plate in case of uniaxial load.
- SCF in Y direction is are much more as compared to the SCF for normal stresses in X direction and shear stress under biaxial load.

References

- [1] Wu, Hwai-Chung, and Bin Mu. "On stress concentrations for isotropic/orthotropic plates and cylinders with a circular hole." *Composites Part B: Engineering* 34.2 (2003): 127-134.
- [2] Adams K. The effect of cutouts on strength of GRP for naval ship hulls. MS Thesis. Department of Materials Science and Engineering, MIT; 1986.
- [3] C. E. Inglis. 1913. Stresses in a Plate Due to the Presence of Cracks and sharp Corners. Transactions of the institution of naval architects. London, England. Engineering. 95: 415.
- [4] G. Kolossoff. 1914. Die Theorie der Elastizitat und die Bedllrfnisse der Festigkeitslehre. Zeitschriftfur Mathematik und Physik. 62: 384.-409.

- [5] H. Neuber. 1937. Kerbspannungslehre, Julius Springer, Berlin, Germany.
- [6] R. E. Peterson. 1966. Stress concentration design factors. New York: John Wiley and Sons.
- [7] K. R. Y. Simha, S. S. Mohapatra. 1998. Stress concentration around irregular holes using complex variable. *Sadhana*. 23, Part 4: 393-412.
- [8] I. Zirka, M. P. Malezhik and I. S. Chernyshenko. 2004. Stress distribution in an orthotropic plate with circular holes under impulsive loading. *International Applied Mechanics*. 40(2).
- [9] Tafreshi. 1995. Numerical analysis of stresses at oblique holes in plates subjected to tension and bending. *Journal of strain analysis*. 30(4): 317-323.
- [10] M. Suneelkumar, p. Alagusundaramoorthy and R.Sundaravadelu. 2007. Ultimate strength of square plate with rectangular opening under axial compression. *Journal of naval architecture and marine engineering*.
- [11] K. Kalita, S. Halder. 2014. Static analysis of isotropic and orthotropic plates with central cutout under transverse loading. *Journal of Institution of Engineers, Series C, Springer Publications*. 95(4): 347-358.
- [12] K. Kalita, D. Shinde, and T. T. Thomas. "Non-dimensional Stress Analysis of an Orthotropic Plate." *Materials Today: Proceedings* 2.4 (2015): 3527-3533,.
- [13] Kalita, Kanak. 2014. Stress concentration mitigation in clamped steel plates. *International Journal of Scientific World*. 2(1): 21-26.
- [14] Kalita, Kanak, Dinesh Shinde, and Salil Halder. "Analysis on Transverse Bending of Rectangular Plate." *Materials Today: Proceedings* 2.4 (2015): 2146-2154.
- [15] F. Darwish, G. Tashtoush, M. Gharaibeh, Stress concentration analysis for countersunk rivet holes in orthotropic plates. *Eur. J. Mech. A* 37, 69–78 (2013).